Nova Scotia Examination: Mathematics 10 *Lessons Learned*



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Purpose of This Document

This document was developed based on an analysis of the results of the Nova Scotia Examination (NSE): Mathematics 10. It is intended to support classroom teachers from Grades 9 to 12 as well as school administrators, regional centre staff, and department staff. The focus of the document is to help educators work through the process of taking in the information provided by the data analysis and see how it can inform lesson design and assessment in the classroom. Topics explored in Lessons Learned are chosen based on the analysis of examination items.

It is suggested that school teams make use of this resource in concert with their school's Item Description Report provided by the Department of Education and Early Childhood Development to all regional centres for education. These reports include student achievement data at the school, regional centre, and provincial level for all questions appearing on the Mathematics 10 examination. By analyzing their own performance on groupings of questions dealing with similar outcomes, schools can identify areas of strength and areas where changes in instruction and/or assessment might be made. This process is designed to foster continued discussions, explorations, and support for mathematics focus at the classroom, school, regional centre, and provincial levels that are all based on valid and reliable data.

This document specifically addresses some of the areas that students across the province found challenging based on provincial examination data. It is essential that teachers consider assessment evidence from a variety of sources to inform the next steps most appropriate for their students. Effective classroom instruction and assessment strategies are responsive to the individual learners within a classroom.

This document highlights those outcomes where students seem to require additional support. It provides some information about student performance on the examination in addition to suggested classroom instruction strategies. Sample assessment items are included for each topic explored.

Lessons Learned – Overview

Provincial assessments and examinations generate information that teachers can use to help inform classroom instruction and assessment. Following the analysis of each assessment or examination, patterns and trends are identified that become the basis for our Lessons Learned documents.

There are four areas that have been identified as the areas of focus for this Lessons Learned document. They are:

- Simplication of exponential expressions
- Factoring polynomials and identifying common factors
- Recognizing the relationship between dimensions of 3-D objects and surface area and volume
- Understanding correlation and correlation coefficients

Each section will address the following questions:

- A. What conclusions can be drawn from the NSE Mathematics 10 analysis?
- B. What are the common student misconceptions or errors?
- C. What are possible next steps in instruction for the class and for individual students?
- D. What are some classroom activities that can help reinforce this concept?

Algebra and Number Outcome 3

AN03 Students will demonstrate an understanding of powers with integral and rational exponents.

A. What conclusions can be drawn from the NSE Mathematics 10 analysis?

When analyzing the provincial examination data, it was noted that students were able to carry out straightforward one step operations involving exponents and exponent laws. Only 8% of students successfully answered a question involving the simplification of an expression requiring the application of multiple exponent laws.

When dealing with patterning to explain an exponent law, only 17% of students were able to use patterning to explain how to determine the value of an expression with a negative exponent.

B. What are the common student misconceptions or errors?

There was confusion between a negative exponent and a negative base. Some students would invert the negative base and drop the negative sign while leaving the exponent intact.

It was also observed that a number of students did not recognize that squaring a negative base yields a positive answer.

Many students were unable to connect the pattern obtained in an exponential sequence (e.g. 2^2 , 2^1 , 2^0 , 2^{-1} , 2^{-2} , ...) to development of exponent laws.

C. What are possible next steps in instruction for the class and for individual students?

Students are introduced to the concept of exponents and exponent laws in Grade 9 for powers with integral bases and whole number exponents through patterning. They also investigate the value of the zero exponent through patterning. It is therefore important that the concept of negative exponents in Grade 10 be explored by students through the use of patterns. It is worth noting that the curriculum document has activities that promote student exploration of this concept; the textbook is weak in this area.

The following are examples of activities that can be used in the classroom to promote the understanding of exponent laws through student exploration.

Ask students to answer the following questions, either through group discussion or on their own.

- 1. a) What does an exponent represent?
 - b) What happens to the value of powers as exponents increase?
 - c) What happens to the value of powers as exponents decrease?
 - d) Based on your observations answering questions b and c, what is the value of 2°?



2. Exploring patterns to understand the value of negative exponents.

Hand the students a chart similar to the one below.

2 ⁶ =	2 ⁰ =	2 ⁻¹ =
2 ⁵ =		2 ⁻² =
2 ⁴ =		2 ⁻³ =
$2^3 = $		2 ⁻⁴ =
2 ² =		2-5 =
2 ¹ =		2-6 =

Have students fill in the values for the powers with positive exponents and the exponent 0. Ask them to complete the following statements:

If the value of the exponent increases by one, the value of the power ______.

If the value of the exponent decreases by one, the value of the power ______.

Once you have discussed the statements, ask students to use the established pattern to fill in the values for the powers with negative exponents. Ask students to share their responses. Display the correct responses as fractions for all students to see. Ask students to share any observations. At this point students are ready to be introduced to the law for negative exponents $a^{-n} = \frac{1}{a^n}, a \neq 0$.

Students will encounter situations where they have to deal with negative bases and negative exponents. It is important for students to build an understanding of the differences that exist between various representations.

Example 1

$$-3^4 = (-1)(3)(3)(3)(3) = -81$$

 $(-3)^4 = (-3)(-3)(-3)(-3) = 81$

Example 2

$$-3^{-4} = (-1)\left(\frac{1}{3^4}\right) = (-1)\left(\frac{1}{(3)(3)(3)(3)}\right) = -\frac{1}{81}$$
$$(-3)^{-4} = \frac{1}{(-3)^4} = \frac{1}{(-3)(-3)(-3)(-3)} = \frac{1}{81}$$

At the end of the exponents section, students should be at ease with simplifying exponential expressions using numbers and variables. They should be able to simplify expressions requiring the application of more than one exponent law. It is important that they also be able to generalise the exponent laws. The following is an example of a series of questions that asks students to generalise their understanding.

- 3. What do you know about x in the following situations?
 - a) 3^x > 1
 b) 3^x <1
 c) 3^x = 1

D. What are some classroom activities that can help reinforce this concept?

$2^{14} = 16\ 384$	$2^9 = 512$	2 ⁴ = 16	$2^{-1} = \frac{1}{2}$	$2^{-6} = \frac{1}{64}$	$2^{-11} = \frac{1}{2048}$
2 ¹³ = 8192	2 ⁸ = 256	$2^3 = 8$	$2^{-2} = \frac{1}{4}$	$2^{-7} = \frac{1}{128}$	$2^{-12} = \frac{1}{4096}$
2 ¹² = 4096	2 ⁷ = 128	$2^2 = 4$	$2^{-3} = \frac{1}{8}$	$2^{-8} = \frac{1}{256}$	$2^{-13} = \frac{1}{8192}$
211 = 2048	$2^6 = 64$	$2^1 = 1$	$2^{-4} = \frac{1}{16}$	$2^{-9} = \frac{1}{512}$	$2^{-14} = \frac{1}{16384}$
2 ¹⁰ = 1024	$2^5 = 32$	$2^{0} = 0$	$2^{-5} = \frac{1}{32}$	$2^{-10} = \frac{1}{1024}$	

Use the table above using the powers of 2 to calculate the following values. Express your answer as a rational number. Do not use a calculator.

4. $\frac{1}{256} \times 16384$	5.	$8192 \times \frac{1}{64}$	6.	$\frac{1}{64} \times \frac{1}{32}$
7. $8 \times \frac{1}{32}$	8.	$\frac{16384 \times \frac{1}{64}}{\frac{1}{8} \times 64}$	9.	$\frac{\frac{1}{4} \times \frac{1}{128}}{\frac{1}{32}}$

Questions where the values of the powers are beyond the calculator's ability to calculate are an excellent way to check the student's understanding of the exponent laws. The following is an example:

- 10. Which value is the greatest?
 - a) 2⁵⁵⁰
 b) 16(2⁴⁰⁰)
 c) 4³⁰⁰
 d) 1024⁵⁰

In this case the students need to transform all of the terms to powers with a common base (likely base 2) in order to compare their value.

Algebra and Number Outcome 5

AN05 Students will be expected to demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially, and symbolically.

A. What conclusions can be drawn from the NSE Mathematics 10 analysis?

When analyzing the provincial examination data, it was noted that students had difficulty associating a concrete or pictorial model to the concept of trinomial factoring. When asked to solve a problem involving a concrete model, only 50% of the students were able to successfully answer the question.

Identifying common factors and factoring difference of squares also seemed to pose some difficulty for students with less than 50% able to answer such questions successfully.

B. What are the common student misconceptions or errors?

It appears that students are not making the connections between the concrete/pictorial models for trinomial factoring and the corresponding symbolic representation.

Examination data seems to show that students are able to carry out straightforward factoring of trinomials and decomposition. Difficulties seem to arise when students are presented with problems that are somewhat unfamiliar yet draw on basic factoring skills. Such questions often ask students to combine or integrate concepts in order to be successful in answering the question.

C. What are possible next steps in instruction for the class and for individual students?

From the very beginning of elementary mathematics, students have been making use of manipulatives. It has been well documented that students learn best when moving from concrete, to pictorial, and then to abstract representations. This is true not only of elementary mathematics but to all mathematics; the use of manipulatives at all levels is important.

When considering factoring, the use of algebra tiles is crucial in the development of the required skills. The use of manipulatives/concrete materials help to provide students with foundational understandings but they also provide deeper understandings as well. Students have used base ten manipulatives throughout their mathematics learning, in particular as a model for multiplication. For this reason it is important that the algebra tiles be available to students during instruction, classroom assessments, and the Nova Scotia Examination. Most students are able to successfully answer the routine factoring and modeling questions that are asked of them on the examination. The difficulties lie with questions that integrate some of these skills or ask students to make connections between the symbolic and the concrete or pictorial representations. It is important to remember to make these connections explicit for students and to revisit the concrete and pictorial representations.



D. What are some classroom activities that can help reinforce this concept?

When working on developing factoring skills, it would be useful to use a model where the students divide a page into three columns. Use the following headings for the columns: concrete representation, pictorial representation, and symbolic representation.



Note the use of the solid black lines that students may use to frame the tiles and/or their drawing. This can help some students with organization as well as with identifying the factors along each side.



Example: Show the product of x + 2 and x + 1.

Once students have become skilled at working with factors and are able to identify common factors, it is important to start introducing activities and problems that are more complex and involve the integration of skills and applied knowledge. The following question is an example that integrates different factoring skills and problem solving.

1. (a) The volume of a rectangular prism can be represented by the polynomial equation $V = 4x^3 - 68x^2 + 288x$. What are one set of expressions that could represent each of the dimensions of the prism?

Here students need to realize that factoring will lead them to the expressions for the dimensions. They may be confused by the x^3 term but should then realize that 4x is a common factor and begin by factoring it out.

(b) If x = 12 cm, determine the exact length, width, and height of the prism.

It is important to present a variety of problems for students to factor that incorporate the different factoring skills addressed in the course. For example:

2. Factor the expressions below.

(a)
$$18m^2 - 2n^2$$
 (b) $7x^3y^2 - 28x^5$

- 3. What factor is common to both $7n^2 + 8n + 1$ and $n^2 1$?
- 4. Pictorially show $(x^2 + 8x + 7) \div (x + 1)$.
- 5. What is the connection between the dimensions of the algebra tile rectangle and the trinomial?

Measurement Outcome 3

M03 Students will be expected to solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including right cones, right cylinders, right prisms, right pyramids, and spheres.

A. What conclusions can be drawn from the NSE Mathematics 10 analysis?

When analyzing the provincial examination data, it was noted that students had some difficulty with identifying the changes that occur to surface area and volume when a dimension is changed in a 3-D object. Only 43% of students were successful in identifying the relationship between the radius of a sphere and its volume.

When solving analysis questions requiring the re-arranging of formulas or solving for an unknown dimension (sometimes requiring use of cube roots), less than 60% of students were successful in determining the correct answer. It is important to note that by their nature, analysis questions are novel and therefore have a higher difficulty level.

B. What are the common student misconceptions or errors?

When presented with a dimension that changes by a set factor, the students assume that the area or volume also changes by that set factor. For example, they do not realize that if the radius of a circle doubles that the area actually quadruples.

When solving more complex problems with surface area and volume of cylinders and spheres, students were mostly able to identify the correct formula and substitute known values. Students had difficulty with the algebraic manipulation required when asked to solve for an unknown dimension. When dealing with formulas involving radius, the students had difficulty with the application of exponent laws (common error: $r \times 5r = 6r$).

C. What are possible next steps in instruction for the class and for individual students?

It is important that students have the opportunity to discover/explore the relationships between dimensions and surface area and volume themselves. This can be done using activities like the following:

Consider the following spheres:



- 1. a) How do the radii compare?
 - b) Calculate the surface area of each of the spheres. How do the two surface areas compare? Why is this?
 - c) Calculate the volume of each of the spheres. How do they compare? Why is this?
 - d) If a sphere has a radius of r, how much larger would its surface area be if its radius were increased to 3r? What would happen to its volume?
 - e) A sphere with radius of 5 cm has a surface area of 314.16 cm². Another sphere has a surface area of 2827.44 cm². What is its radius?

Students need to be exposed to a wide range of questions, some of which will be challenging, especially those requiring some algebraic manipulation to isolate unknown variables. Since the Measurement unit is taught prior to the Algebra and Number unit it is important to realize that students have not had exposure to working with cube roots; this will have to be taught when dealing with some questions involving 3-D objects. It will also be important to re-visit some measurement questions after having taught the section on exponents and roots in the Algebra and Number unit.

D. What are some classroom activities that can help reinforce this concept?

- 2. A concrete column in a parkade is cylindrical. The column is 10 m high with a diameter of 3.5 m.
 - a) What is the volume of the concrete in the column?
 - b) Suppose the concrete was made into a cube, what would be the dimensions of the cube?
- 3. A study shows that the consumers think the diameter of a large can of coffee is too wide. The study suggests that a narrower can would increase sales. The original can has a diameter of 20 cm and a height of 18 cm. Suppose the diameter of the can is decreased by 20% without changing the volume. What is the height of the new can?
- 4. Take a standard piece of paper (8.5 in. x 11 in.). Which has a greater volume, the piece of paper rolled lengthwise or the piece of paper rolled widthwise? Justify your answer with calculations.

- 5. The radius of a sphere is increased from 3 inches to 5 inches. What is the percentage increase in the volume of the sphere?
- 6. A cord of firewood is 128 cubic feet. Joe has 3 storage bins that measure 8 feet by 6 feet by 4 feet. How many cords of firewood can he store?
- 7. Philip made fudge that filled a 20 cm by 21 cm by 3 cm pan.
 - a) What is the volume of the fudge?
 - b) Philip shares the fudge with 30 people in class, how could Philip cut the fudge so that each person gets an equal size piece?
- 8. A spolight is positioned directly above a performer. If the surface area of the cone of light was approximately 500 square feet and the diameter was 12 feet, how high up above the stage is the spotlight?

Relations and Functions Outcome 7

RF07 Students will be expected to determine the equation of a linear relation to solve problems given a graph, a point and the slope, two points, and a point and the equation of a parallel or perpendicular line.

A. What conclusions can be drawn from the NSE Mathematics 10 analysis?

When analyzing the provincial examination data, it was noted that students had difficulty with the concept of the correlation coefficient. When students were asked to identify the correlation coefficient of data in a straight line or estimate the correlation coefficient for a group of relatively tightly knit set of data points, less than 60% of students were able to successfully answer the question.

B. What are the common student misconceptions or errors?

When presented with data in a straight line, the most common error made by students was confusing the slope of the line with the correlation coefficient for the data.

When presented with a scatter plot and asked to estimate the correlation coefficient, the students seemed to be able to distinguish between a positive and a negative correlation (likely by drawing a parallel to slope) but did not distinguish between strong correlation values and weak correlation values.

C. What are possible next steps in instruction for the class and for individual students?

Correlation Coefficient: (linear relationships)	A number that describes how well a line models the data. The closer to 1 or -1 the coefficient is, the stronger the correlation. A correlation that is positive means that the line has a positive slope. A correlation that is negative means that the line has a negative slope.
	(Mathematical Modeling Book 1, p. 196)

It is important to give the students a good sense of what correlations are and why they are useful in statistics. At this point, it would be good to talk about mathematical models and how they are used in various industries. This is an opportune time to connect the mathematics we study to real-world applications. From epidimiology to insurance rates, mathematical models and correlations are used to make predictions based on observed trends in data.

While it is important to do activities with students that get them to calculate correlation coefficient values with technology, it is more important that the students understand the big ideas related to this topic.

- Correlations range in value from -1 to +1. The closer a value is to the extremes of this range, the stronger the correlation is between the two variables.
- Positive correlation values indicate a positive slope and negative correlation values indicate a negative slope. In this case a negative value does not indicate a "bad" correlation, in fact a -1 correlation is just as strong as a +1 correlation.

D. What are some classroom activities that can help reinforce this concept?

1. The following graphs represent the relationship between two variables. For each of the graphs, identify whether the correlation is strong or weak, and whether it is positive or negative. Use a sentence to describe the relationship between the two variables.



2. Arrange the graphs from strongest to weakest correlation.



Note: Some students will order the graphs from strongest positive correlation to strongest negative correlation. It is important to intervene with these students to reinforce the notion that the sign attached to the correlation coefficient is not an indication of strength, but rather that the absolute value of the correlation is the indication of strength.

Additional problems and data sets related to linear correlations are available in *Mathematical Modeling Book 1,* pages 190-198.

Appendix A Overview of the Nova Scotia Examination: Mathematics 10

The NSE Mathematics 10 provides information about mathematics for each student and complements assessment data collected in the classroom. This assessment is administered at the end of the Mathematics 10 course and is designed to provide detailed information regarding each student's progress in achieving the course's curriculum outcomes. Information from this examination can be used by teachers to inform their instruction and assessment practices as well as to provide support for their students.

The design of the examination includes the following:

- items that are aligned to reflect the curriculum outcomes from the Mathematics 10 course
- items in selected-response and constructed-response formats
- items providing a broad range of challenge, thereby providing information about a range of individual student performance.

Cognitive level of questions

Below are the percentage ranges of questions at each of the cognitive levels in the NSE Mathematics 10:

Knowledge	20–30%
Application	55–65%
Analysis	15–25%

Knowledge questions may require students to recall or recognize information, names, definitions, or steps in a procedure.

Application questions may require some degree of comprehension and students will have to apply their mathematical knowledge to answer correctly.

Analysis questions require students to go beyond comprehension and application to higher order thinking skills, such as generalizations and problem solving.

The NSE Mathematics 10 Information Guide, available at <u>https://plans.ednet.ns.ca</u>, contains additional information on the levels of questions.

It is recommended to use similar question distribution in the creation of well balanced assessment tools.

Appendix B Performance Levels

Student performance on the NSE Mathematics 10 is reported overall as one of four performance levels. Below are the descriptions for each of the levels as they apprear on the student reports. It is important that the language used in the performance level descriptors are based on general observations of what students in each of the level are able to achieve in relation to the outcomes. Through classroom observation and assessment teachers can further identify individual strengths and areas of need.

Level 1 (below the expectation for end of grade 10)

Students at level 1 are generally able to solve problems that are straightforward or where the method to solve the problem is suggested to the student. They are able to solve previously learned routine problems. They have difficulty understanding and using grade-level mathematical vocabulary. They may be able to pictorially and concretely represent/interpret a concept.

Level 2 (approaching the expectation for end of grade 10)

Students at level 2 are able, independently or with prompting, to solve some clearly described problems that require recall of information, recognition of simple patterns, and use of simple procedures. They have success solving straightforward problems in familiar contexts or when a strategy is suggested. They understand and use some grade-level mathematical vocabulary but may confuse meanings and terms. They are able to pictorially, concretely, and contextually represent/interpret a concept and can interpret and represent some mathematical concepts in symbolic form.

Level 3 (at the expectation for end of grade 10)

Students at level 3 have success solving multi-step problems in unfamiliar contexts. They perform number operations $(+, -, \times, \div)$ with confidence and ease, but may make minor errors. They understand and use grade-level mathematical vocabulary and notation. They are able to pictorially, concretely, contextually, and symbolically represent/interpret a concept.

Level 4 (above the expectation for end of grade 10)

Students at level 4 have success in problem solving situations and is able to solve novel and complex problems. They perform number operations $(+, -, \times, \div)$ with confidence and ease. They are consistent when using grade-level mathematical terminology and notation. They are consistently able to use all representations with ease to represent/interpret a concept.

NOTE: By "expectation", it is meant the expecation of student performance in relation to the examination and not in relation to the curriculum outcomes as there are no questions on the examination that go beyond what is expected from the curriculum outcomes.

Appendix C Answers to questions

AN03

- 1. a) An exponent represents the number of times the base is multiplied by itself.
 - b) If the base is greater than 1, the power will increase in value as the exponent increases.
 - c) If the base is greater than 1, the power will decrease in value as the exponent decreases.
- 2.

2 ⁶ = <mark>64</mark>	2 [°] = 1	$2^{-1} = \frac{1}{2}$
2 ⁵ = 32		$2^{-2} = \frac{1}{4}$
2 ⁴ = 16		$2^{-3} = \frac{1}{8}$
2 ³ = 8		$2^{-4} = \frac{1}{16}$
2 ² = 4		$2^{-5} = \frac{1}{32}$
2 ¹ = 2		$2^{-6} = \frac{1}{64}$

If the value of the exponent increases by one, the value of the power is multiplied by the base once.

If the value of the exponenet decreases by one, the value of the power is divided by the base once.

3. a)
$$x > 0$$
 b) $x < 0$ c) $x = 0$

4. 64 5. 128 6.
$$\frac{1}{2048}$$
 7. $\frac{1}{4}$ 8. 32 9. $\frac{1}{16}$

10. option c has the greatest value

AN05

1. a) Answers will vary, one correct set of dimensions is 4x, x - 9, and x - 8.

b) The dimensions of the prism are 48 cm by 3 cm by 4 cm.

2. a)
$$2(3m + n)(3m - n)$$
 b) $7x^{3}(y - 2x)(y + 2x)$

3. *n* + 1

4. x + 7 (pictures will vary)

- 1. a) The radius of the larger sphere is twice that of the smaller sphere.
 - b) The surface area of the small sphere is 36π and that of the large sphere is 144π . The surface area of the large sphere is 4 times that of the small sphere. Since the radius is squared in the formula, any change will result in a squaring of that change in the surface area. For example, if one triples the radius, the surface area will increase by a factor of 3^2 or 9.
 - c) The volume of the small sphere is 36π and that of the large sphere is 288π . The volume of the large sphere is 8 times that of the small sphere. Since the radius is cubed in the formula, any change will result in a cubing of that change in the volume. For example, if one triples the radius, the volume will increase by a factor of 3^3 or 27.
 - d) If the radius were increased to 3*r*, the surface area of the sphere would increase by a factor of 9 and its volume by a factor of 27.
 - e) Since 2827.44 cm² is 9 times larger than 314.16 cm², the radius of the second sphere is 15 cm.
- 2. a) 30.625π m³ or 96.2 m³ b) the side length of the cube would be 4.58 m
- 3. The new height would be 28.125 cm.
- 4. Rolled lengthwise: V = 81.85 in.³ Rolled widthwise: V = 63.24 in.³
- 5.363%
- 6. 4.5 cords of wood
- 7. a) 1260 cm³ b) Phillip should cut the fudge into 7 cm by 2 cm pieces.
- 8. 19.63 ft.

RF07

1. Strong negative correlation. The lighter the bodyweight of the jumper, the higher they will jump.

Strong positive correlation. The heavier the bodyweight of the lifter, the more they will be able to lift.

Weak (or no) correlation. The bodyweight of a dart player has no impact on the score when three darts are thrown.

2. b, a, d, c, e