LESSONS LEARNED

from Nova Scotia Assessment: Mathematics in Grade 8

Module 1 – Fractions

“We assess students not merely to evaluate them, but to improve the entire process of teaching and learning.”
- Douglas B, Reeves, Making Standards Work, 2004

NOVA SCOTIA

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Purpose of this Document

This Lessons Learned document was developed based on an analysis of the Item Description Reports for the Nova Scotia Assessment: Mathematics in Grade 8 (M8). This document is intended to support all classroom teachers at grades 6 - 8, and administrators at the school, region, and provincial levels. The focus of the document is to help educators work through the process of taking in the information provided by the data analysis and see how it can inform lesson design and assessment in the classroom. Topics explored in Lessons Learned are chosen based on the analysis of assessment items.

It is suggested that school teams make use of this resource in concert with their school's Item Description Report provided by the Department of Education and Early Childhood Development to all regional centres for education. These reports include student achievement data at the school, regional centre, and provincial level for all questions appearing on the Mathematics in Grade 8 Assessment. By analysing their own performance on groupings of questions dealing with similar outcomes, schools can identify areas of strength and areas where changes in instruction and/or assessment might be made. This process is designed to foster continued discussions, explorations, and support for mathematics focus at the classroom, school, regional centre, and provincial levels that are all based on valid and reliable data.

This document specifically addresses some of the areas that students across the province found challenging based on provincial assessment data. It is essential that teachers consider assessment evidence from a variety of sources to inform the next steps most appropriate for their students. Effective classroom instruction and assessment strategies are responsive to the individual learners within a classroom.

This document highlights those outcomes where students seem to require additional support. It provides some information about student performance on the assessment in addition to suggested classroom instruction strategies. Sample assessment items are included for each topic explored.

Lessons Learned Overview

Provincial assessments and examinations generate information that teachers can use to help inform classroom instruction and assessment. Following the analysis of each assessment or examination, patterns and trends are identified that become the basis for our Lessons Learned documents.

There are two areas that have been identified as the areas of focus for this document:

- Equivalent representations of fractions
- Misapplication of computational algorithms with fractions

Each section will include an overview of why this topic is an area of focus by describing student errors and misconceptions. It will outline possible strategies to support student learning on this topic as well as provide sample lesson activities and assessment questions. At the end of each section, a list of supporting resources is provided to further professional learning and student learning.

What conclusions can be drawn from the NSA: Mathematics in Grade 8?

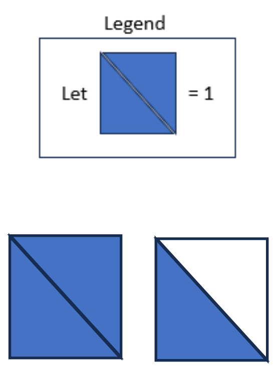
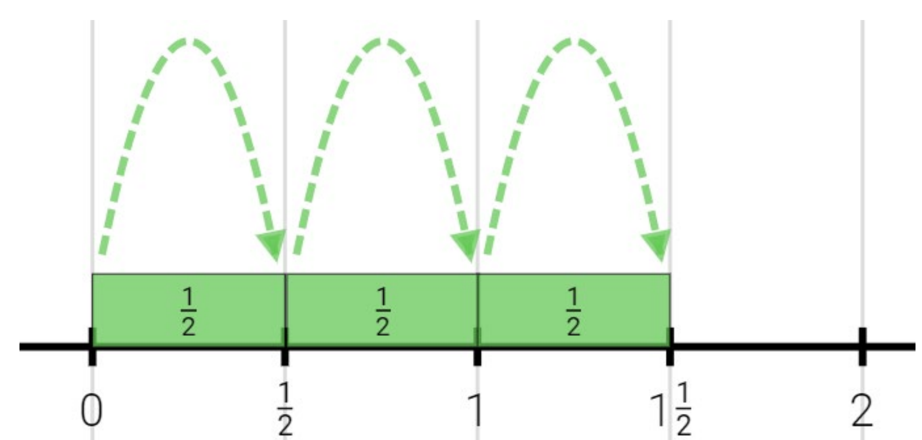
Upon close analysis of the responses collected from the NSA: Mathematics 8 assessment (M8), nearly half of our NS students are having difficulty correctly answering questions that include operations with fractions. From the outset of their journey in calculating values with fractions, students often find it challenging to align their understanding of mathematical operations with the impact these operations can have on fractional values. When working with whole numbers, the same students have demonstrated knowledge of the four operations but when working with fractional values, they begin to misapply computational algorithms they have seen during instruction or illustrated in textbooks. This is often the same roadblock students encounter when asked application or analysis questions. Even if they can translate the information given in an expression or equation, the mechanics of working with the fractional values makes resolution an even more daunting task.

When working on these problems, students need to make use of the resources and tools at their disposal. Manipulatives and pictorial representations should be common place during instruction and classroom activities. These representations can be used as entry points, reminders, and benchmarks to confirm the reasonableness of answers. Working with varied representations also helps students to learn how to use their reasoning skills to select appropriate problem solving strategies.

<table>
<tr><th colspan="3">Operations on Fractions</th></tr>
<tr><th colspan="2">Alignment to Previous Outcomes</th><th>Related Outcomes</th></tr>
<tr><td rowspan="2">6N04
Students will be expected to relate improper fractions to mixed numbers and mixed numbers to improper fractions.</td><td>7N05
Students will be expected to demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences).</td><td rowspan="2">8N06
Students will be expected to demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically.</td></tr>
<tr><td>7N07
Students will be expected to compare, order, and position positive fractions, positive decimals (to thousandths), and whole numbers by using benchmarks, place value, and equivalent fractions and/or decimals.</td></tr>
</table>

Equivalent Representations:

Data from the NSA Mathematics in Grade 8 shows that students from across the province demonstrate difficulties with items that require them to move from one representation to an equivalent representation (improper fraction to mixed number, fractions of different denominators, etc.) to solve problems.

Misconceptions/Errors in Student Work	Possible Next Steps in the Classroom
When students are asked "What value is represented by this picture?";  i) students may answer like this: $\frac{2}{3}$ This is an inversion of the correct answer; students may believe that a fraction should always be between 0 and 1. *Reflection Questions:* - What is the whole? - Should the answer be 'greater than' or 'less than' one whole?	Each of the different type of responses give us clues as to the root of the misconception. Before planning next steps, it is important to understand what the student understands well and tailoring their learning from their starting point. Regardless of what the student understands, using various forms of representation can only further their understanding and / or add flexibility to their thinking. - Students that answer $\frac{2}{3}$ may require some further examples of part-whole situations with real world context; to reinforce the understanding that a fraction can describe an amount greater than 1. They have clearly identified the values that make up the correct answer but have inverted them to satisfy the condition that fractions are supposed to be between 0 and 1. Walk through the collection of information that shows the values 2 and 3. Then extend to the notion that a numerator larger than the denominator can be correct depending on the defined whole. Visuals are key! Number lines, pictorial representations, and / or manipulatives. 

ii) students may answer like this:

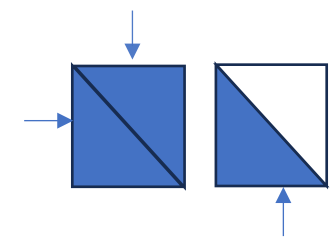
$$\frac{1}{2}$$

This is often an answer because students are focused on the part and have forgotten or ignored the whole.

Reflection Questions:

- Does this answer represent the entire image?
- Is there more to the image?

- Students that answer $\frac{1}{2}$ could be supported by using guiding questions to get them to express how they know there is a half represented in the image. Isolate the whole and use their answer to construct the whole. Show that a whole and a half can be represented with an improper fraction, via counting unit fractions.

<table>
<tr><td colspan="2"></td></tr>
<tr><td>$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{3}{2}$</td><td>$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=1+\frac{1}{2}$</td></tr>
<tr><td>- One half
- Two halves
- Three halves

Context: 3 half glasses of water for 3 people</td><td>- One half
- One whole
- One whole and a half

Context: Recipe says bake for $1\frac{1}{2}$ hours</td></tr>
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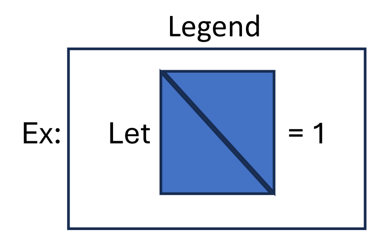
<td>
iii) students may answer like this:
$$\frac{3}{4}$$
This is often an answer because students are considering all the space as the whole but still correctly identifying the coloured portions, and thus correctly identifying the numerator.

Reflection Questions:
- What is the whole in this image?
- How many equal parts are there in one whole?
</td>
<td>
- Students that answer $\frac{3}{4}$ could be supported by ensuring that the definition of the whole is clear. Talk about which visual cues can help them determine the whole in other situations too. Use the diagram to show both improper fractions, as well as mixed numbers. Establish a classroom routine of asking “What is the whole?” whenever there is a new context or problem. Make it common practice on your own assessments to define the whole, within the stem of the item, as a legend:



- Students that answer correctly have a solid understanding of translating from the pictorial to the symbolic representation of the values greater than 1. It is important that you expand on the correct answer and discuss the strategies that lead to it and other equivalent representations ie: mixed numbers, decimals, and percentages. Going deeper with students can only strengthen comprehension and increase their flexibility in thinking, which is key to problem solving. Having stronger students share can improve communication skills and prompt other students to see diverse types of solutions.
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Activities to Support Lesson Planning

Grade 6

Which one doesn't belong?
What do you notice?

These types of warm ups can often be presented with little to no explanation, other than "What do you notice?" and then managing the conversation. Students will benefit from having each part of the grid named so they can quickly and easily identify which they would like to discuss. In these examples, they are labelled as you would the quadrants of a cartesian plane. This is just an idea to interleave an additional concept into a classroom routine. Feel free to label them how ever you would like.

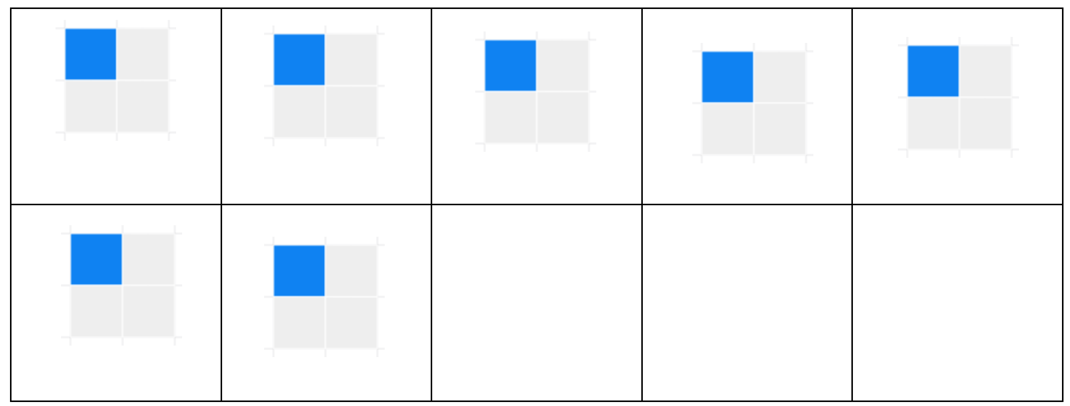
ii) $\frac{3}{2}$	**i)** $\frac{4}{3}$
iii) $1\frac{1}{3}$	**iv)** $\frac{8}{6}$

ii) $\frac{7}{5}$	**i)** $1\frac{2}{5}$
iii) $\frac{14}{10}$	**iv)** $\frac{3}{2}$

Grade 7

Counting by unit fractions with fraction images

Can your students also represent this as a mixed number?
How might the organization of the images support this process?



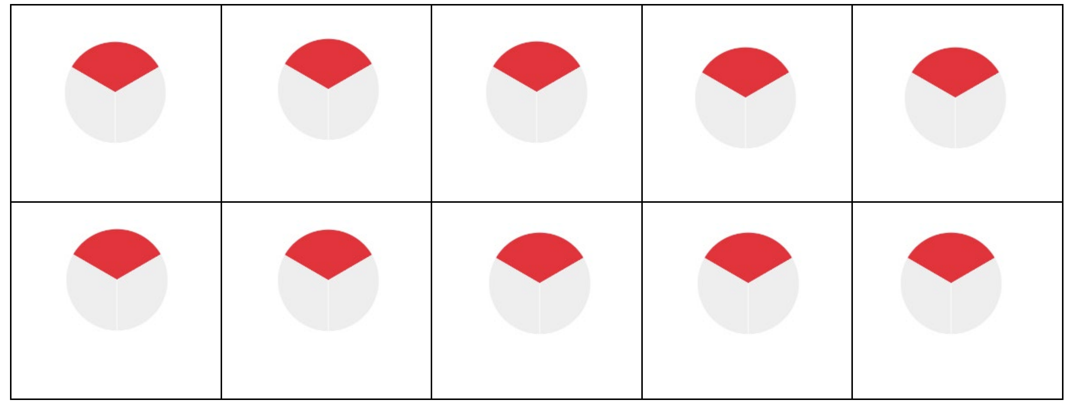
Research suggests the one piece of consolidating a student's understanding of fractions is counting by unit fractions. This familiar layout should allow the student to quickly identify how many unit fractions are displayed, therefore creating a link between the repeated addition of 7 fourths and the improper fraction $\frac{7}{4}$.

Grade 8

Number strings:
Distributive property / Finding Friendly Numbers

Number Strings are a structured sequence of related math problems designed to help students develop fluency, flexibility, and strategic thinking with numbers. By carefully selecting problems that build on prior knowledge, teachers can guide discussions that encourage students to explore patterns, make connections, and refine their problem-solving strategies. This routine fosters mathematical discourse, allowing students to articulate their reasoning, select efficient strategies, and deepen their understanding of number relationships.

i)	$5\frac{1}{3} \times 3$	$5\frac{1}{3} = 5 + \frac{1}{3}$ **Why is that important?** **Did you distribute?** **3 and $\frac{1}{3}$ are friendly numbers – Why?**
ii)	$5\frac{1}{3} \times 6 \times 2$	**We know 3 and $\frac{1}{3}$ are friendly numbers but we don't have a 3.** **Can we factor out a 3? Can we use the previous solution?** $= 5\frac{1}{3} \times 3 \times 2 \times 2$ $= (15 + 1) \times 2 \times 2$ $= 8 \times 2 \times 2 \times 2$ $= 8 \times 8$

Some guiding notes for these two examples: Top i) only value that is a reduced form of one of the other values (iv) ii) only value that is not equivalent to the others, greater than the others iii) only mixed number iv) only value not reduced to simplest form Bottom i) only mixed number ii) only value this a reduced form of one of the other values (iii) iii) only value not reduced to simplest form iv) only value that is not equivalent to the others, greater than the others	 Students should immediately recognize this as 10 things. In this example, 10 thirds or $\frac{10}{3}$. Does this layout lend itself to translating from improper fractions to mixed numbers? Why does this work? Would it still work if the fractions were each different?	**iii)**	$5\frac{1}{3} \times 3 \times \frac{1}{4}$	**How do the solutions from problems i) and ii) help us here?** **How does the final answer change when we multiply by $\frac{1}{4}$, instead of 4?**
		vi)	$5\frac{1}{3} \times \frac{3}{4}$	**How does iii) help us solve vi)?** $\frac{3}{4} = \frac{3}{1} \times \frac{1}{4}$
		Other examples to develop	$3 \times \frac{1}{5} \times 10$ $4 \times 1\frac{1}{5} \times 10$	**Do any of the previous computations help us solve this multiplication?** **What strategies might we rely on? (doubling, factors, friendly numbers)** **What other property is important here?**

Misapplication of Computational Algorithms with Fractions:

Data from the NSA Mathematics in Grade 8 shows that students from across the province demonstrate difficulties with items that require them to complete one or more operations on fractional values. The following highlights each operation individually, but items on the assessment would have also included problems with multiple operations. Many calculations require a certain amount of fluidity and ease with regards to changing between equivalent representations (see previous sections).

Misconceptions/Errors in Student Work

Although 8N06 is specific to multiplication and division with fractions, it is essential that addition and subtraction continue to be interwoven into student learning in grade 8.

Be sure to reinforce the relationships between addition and multiplication (repeated addition), as well as subtraction and division (repeated subtraction).

Here are examples of misconceptions/errors for all four operations:

Multiplication

Students may do this:

$$\frac{2}{3} \times \frac{6}{5} = ?$$

$$= \frac{2}{3} \times \frac{6}{5}$$

$$= \frac{2(5) \times 6(3)}{(3 \times 5)}$$

$$= \frac{180}{15}$$

$$= 12$$

Possible Next Steps in the Classroom

Because of previous knowledge students will often try to use rules they are familiar with and make changes to those rules to match the notation, without ensuring the changes are mathematical in nature. For example, finding common denominators to multiply two fractions.

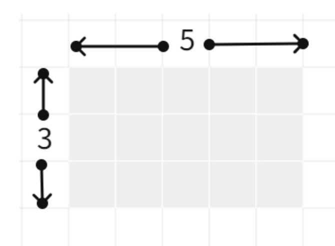
Real world contexts will help ground student understanding and make new strategies more familiar and easier to connect with.

To support student learning be sure that it is common practice to use multiple representations (concrete, pictorial and symbolic) when working with fractions.

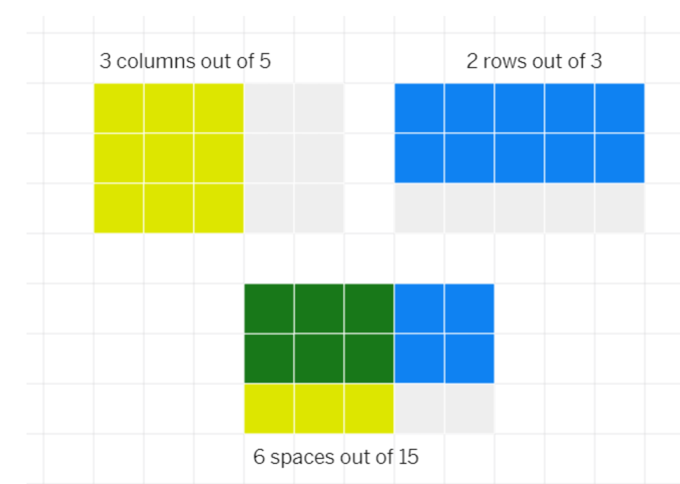
Multiplication

Use a grid model to illustrate multiplication of two fractions. The grid should have dimensions equal to the values of the denominators of each fraction. Once the grid is made, populate it with colours, designs, or counters to indicate each fraction. The overlap is the product (see below):

Bring out the relationship between multiplication and the word 'of'. The expression $\frac{2}{3}$ of $\frac{3}{5}$ naturally highlights the overlap shown in the grid model.



$$\frac{2}{3} \times \frac{3}{5}$$



This is often an answer because students are trying to create common denominators, as they would have if they were calculating a sum.

Reflection Questions:

- When we multiply is the product always greater?
- Is the answer reasonable?

"Once students understand how to multiply fractions, it is possible to revisit the concept of creating equivalent fractions […]."

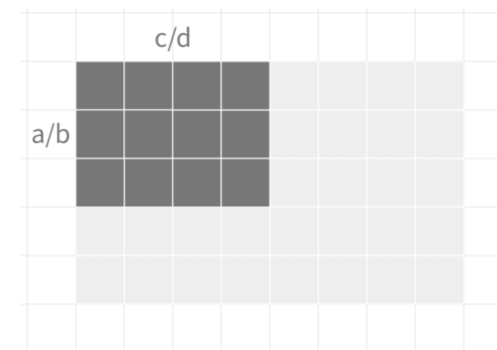
- Dr. Marion Small, *Big Ideas from Dr. Small (Grades 4-8)*, Nelson (2009)

Once the students have had the opportunity to work with several examples, a consolidation activity could be:

- generalize the grid for any two fraction factors

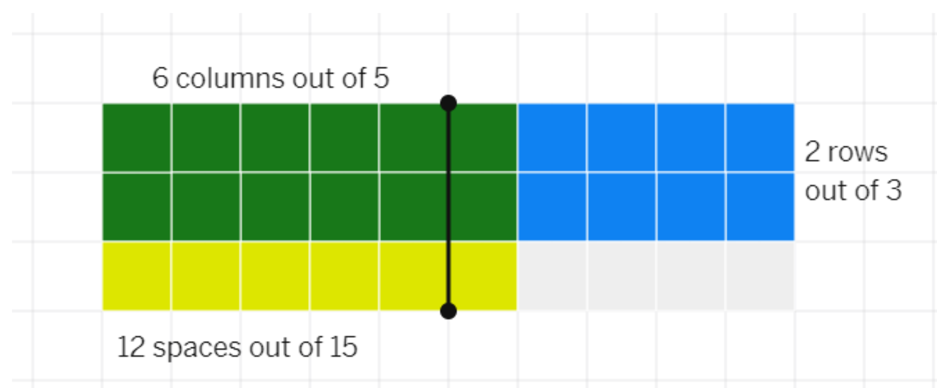
Dimensions of the grid: (b x d)

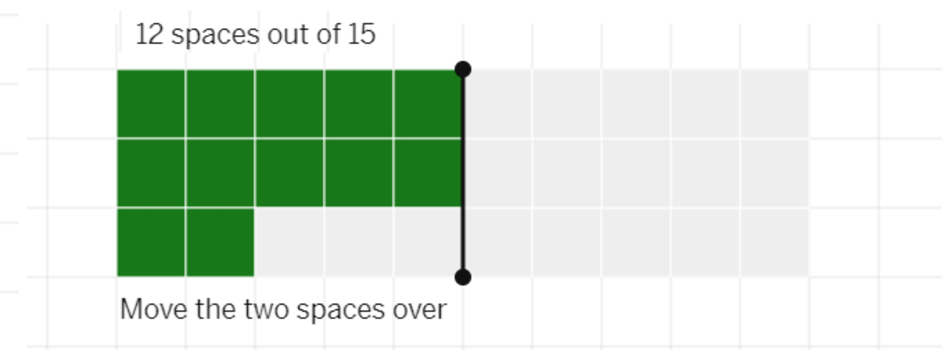
Product: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$



- show how the grid could be used for mixed numbers and improper fractions.

Calculate $\frac{2}{3} \times \frac{6}{5}$;





Answer: $\frac{2}{3} \times \frac{6}{5} = \frac{12}{15}$

This grid shows that one whole is defined by 15 spaces (product of the denominators).
Why are there two grids? (one fraction is greater than one)
What do you notice about the number of green spaces? (Equal to the product of the numerators)
What do you notice about the products of the two examples? (the first one is half of the second)

Division

Students may do this:

$$\frac{3}{4} \div \frac{1}{2} = ?$$

$$= \frac{3}{4} \div \frac{1}{2}$$

$$= \frac{3}{4} \times \frac{1}{2}$$

$$= \frac{3}{8}$$

This is often an answer because students are trying to recall the shortcut 'keep, switch and flip', but have missed the last step. These types of processes reinforce bad math habits and the misconception that mathematics is simply memorizing a series of rules or steps.

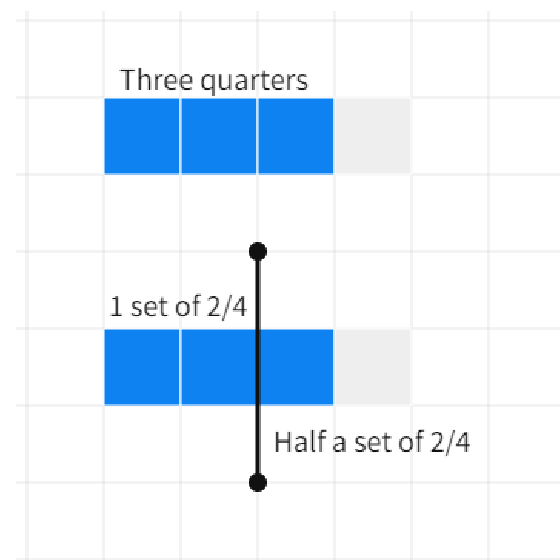
Reflection Questions:

- Is the left-hand side still equivalent to the right-hand side? Why or why not?

Division

Since multiplication and division are inverse operations, the grid model could easily be used to illustrate a division sentence. To diversify your use of visuals and to help more students find a link to the concepts, consider trying fraction strips or Cuisinaire rods.

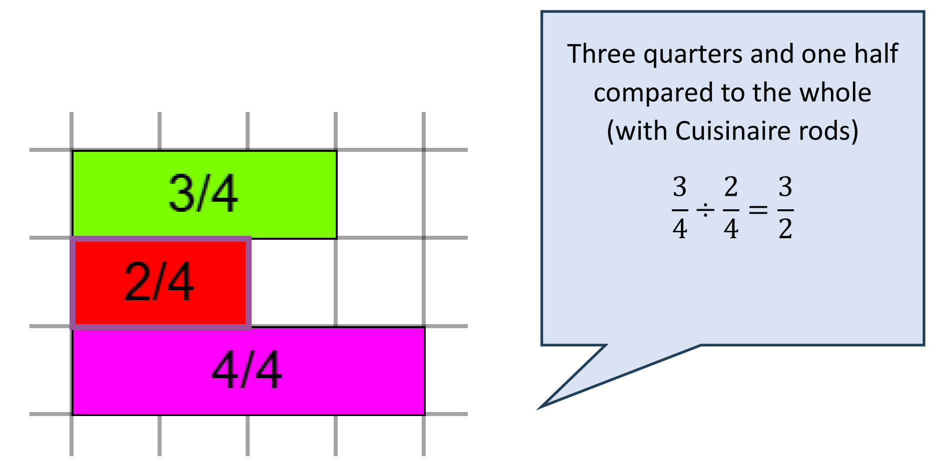
Ex:



$$\frac{3}{4} \div \frac{1}{2}$$

$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \div \frac{2}{4}$$

$$= 1\frac{1}{2}$$



Sometimes the quotient is easily seen visually but once students have more experience calculating quotients, ask the students if they recognize any patterns. Avoid giving them shortcuts like Keep, Switch, Flip. Help students to understand how many groups of $\frac{1}{2}$ do you have in $\frac{3}{4}$. You have 1 whole group and a $\frac{1}{2}$ of a group. Taking it back to the idea of division as groups containing a predetermined amount.

Addition

Students may do this:

$$1\frac{3}{5} + 2\frac{1}{3} = ?$$

$$1\frac{3}{5} + 2\frac{1}{3} = 1 + 2 + \frac{3+1}{5+3} = 3\frac{4}{8} = 3\frac{1}{2}$$

Addition

Adding mixed numbers is simply adding two whole numbers, adding two proper fractions, and finally adding a whole and a proper fraction together. Based on the data from NSA M3 and M6, students are apt at basic addition of positive integers. Therefore, the meat of this misconception is the misapplication of the algorithm for adding two proper fractions.

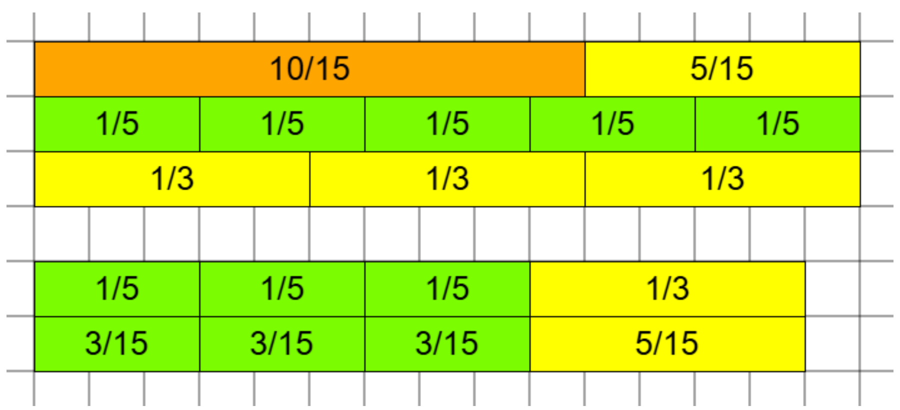
Cuisenaire rods, grids, and other fraction manipulatives should be common place in the classroom so there is always an entry point for every level of student.

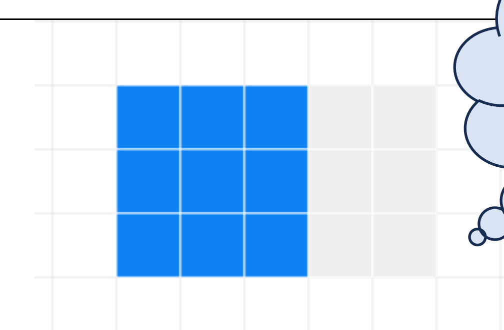
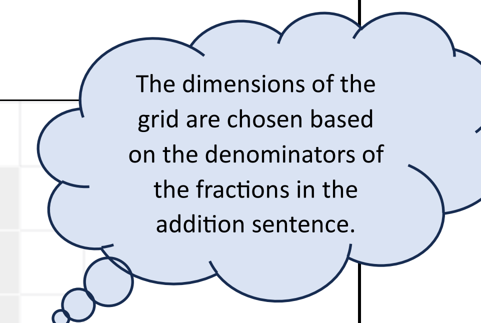
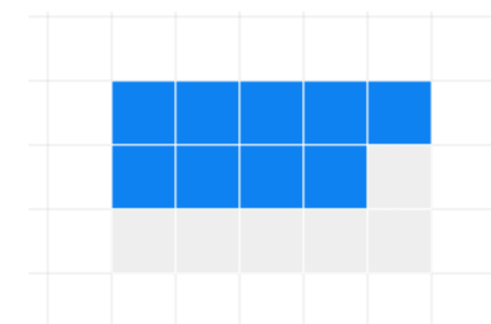
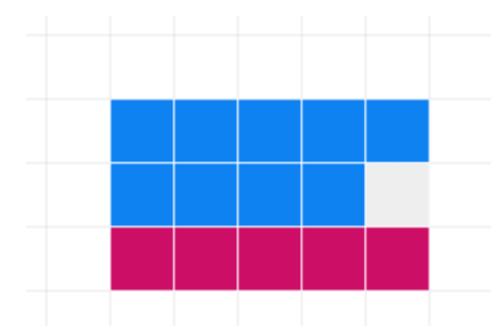
This is often an answer because students are applying the algorithm learned for multiplication and using it to calculate a sum.
Students have correctly identified and added the whole components of the mixed number, so be sure to praise this correct piece.

Reflection Questions:

- What do we like and want to keep from this strategy?
- Is the meaning of the denominator preserved?

Visual representation of $\frac{3}{5}+\frac{1}{3}$ using Cuisinaire rods:



Add the whole parts first, then the fractions. (commutative property)	$1\frac{3}{5}+2\frac{1}{3}=1+\frac{3}{5}+2+\frac{1}{3}=1+2+\frac{3}{5}+\frac{1}{3}$
The sum of the wholes is: **3**	
Represent: $\frac{3}{5}$ 3 of the 5 columns	 
Reorganize: $\frac{3}{5}$ Reorganize the 3 coloured tiles from the third row	
Add: $\frac{1}{3}$ 1 of the 3 rows	
The sum of the fractions is: $\frac{14}{15}$	

Subtraction

Students may do this:

$$4\frac{1}{3} - 1\frac{4}{5} = ?$$

$$4\frac{1}{3} - 1\frac{4}{5} = \frac{5}{3} - \frac{5}{5} = \frac{5-5}{3-5} = -\frac{0}{2} = 0$$

This is often an answer because students are trying to apply the algorithm for multiplication to calculate the difference.

Reflection Questions:

- Is the answer reasonable?
- Where do the halves come from?

Solution :

Count up from $1\frac{4}{5}$ to 2 :

$2 - 1\frac{4}{5} = \frac{10}{5} - \frac{9}{5} = \frac{1}{5}$

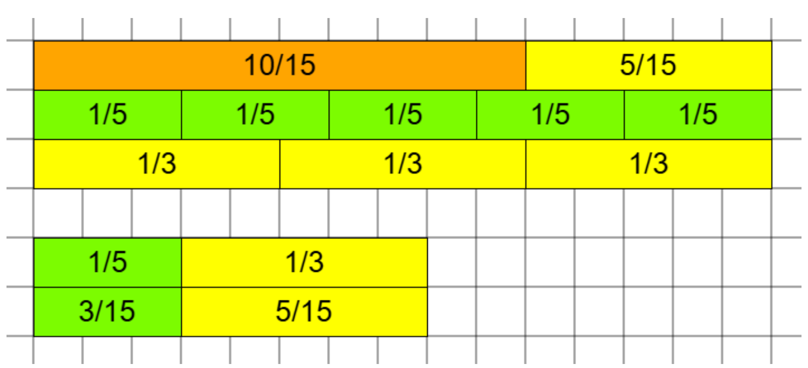
Count up from 2 to 4 :

$4 - 2 = 2$

Count up from 4 to $4\frac{1}{3}$:

$4\frac{1}{3} - 4 = \frac{1}{3}$

Sum: $\frac{1}{5} + 2 + \frac{1}{3} = 2 + \left(\frac{1}{5} + \frac{1}{3}\right) = 2\frac{8}{15}$



Cuisinaire rods

$$Sum\,(wholes) + Sum\,(fractions) = 3 + \frac{14}{15} = 3\frac{14}{15}$$

Subtraction

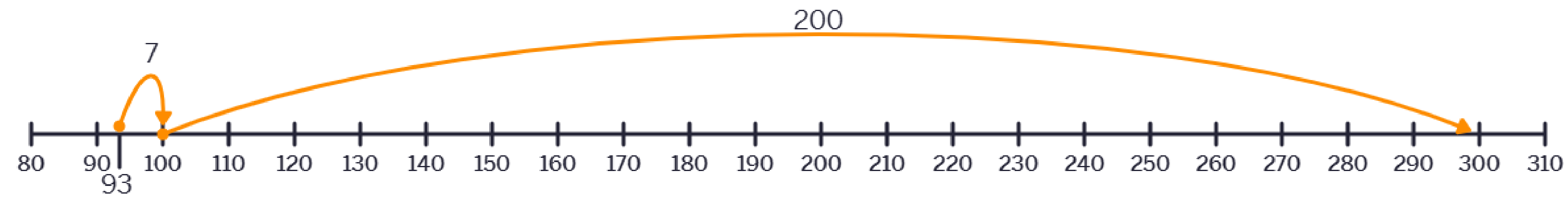
Subtracting a fraction from a whole number or two mixed numbers can be a little more involved. Consider other fluency strategies, like counting up, in order to make connections between whole number subtraction and subtraction with fractional values.

Start with something like: **300 – 93 = ?**

Count up from 93 to 100:
(the next highest benchmark)

Count up from 100 to 300:
(the next highest benchmark)

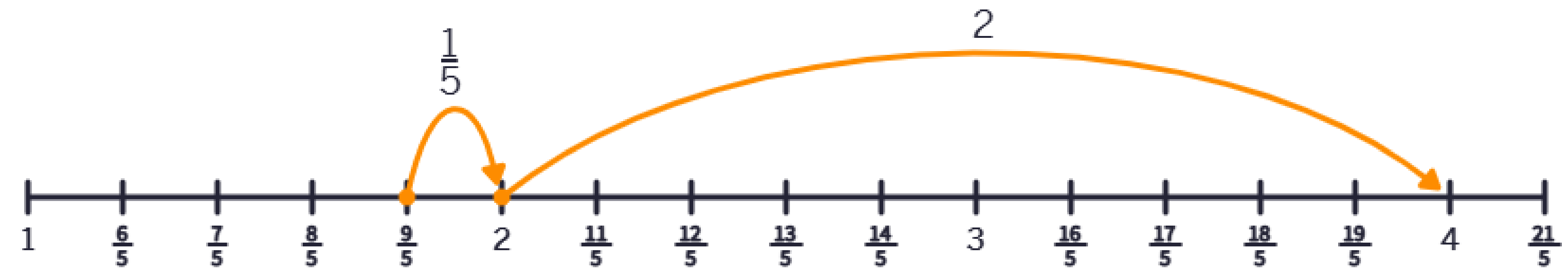
Altogether:
300 – 93 = 207



How could this apply to:
$4 - 1\frac{4}{5} = ?$

Count up:
- from $1\frac{4}{5}$ to 2;
- then from 2 to 4:

Altogether:
$$4 - 1\frac{4}{5} = 2\frac{1}{5}$$



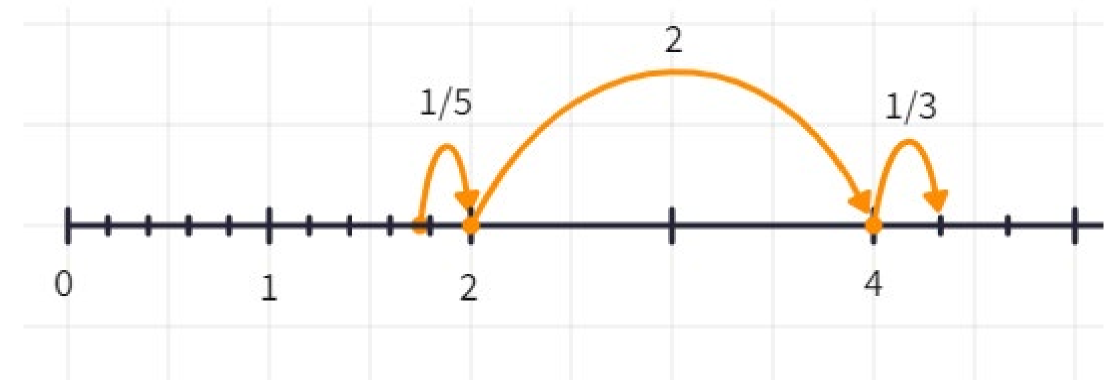
"Armed only with rules, students have no means of assessing their results to see if they make sense. Surface mastery of rules in the short term is quickly lost as the myriad of rules become meaningless when mixed together."

- Bobby Ojose, Assistant Professor of Mathematics Education at Youngstown State University (2015)

Now extend this to the subtraction of a mixed number from another mixed number: $\mathbf{4\frac{1}{3} - 1\frac{4}{5}}$

Count up:

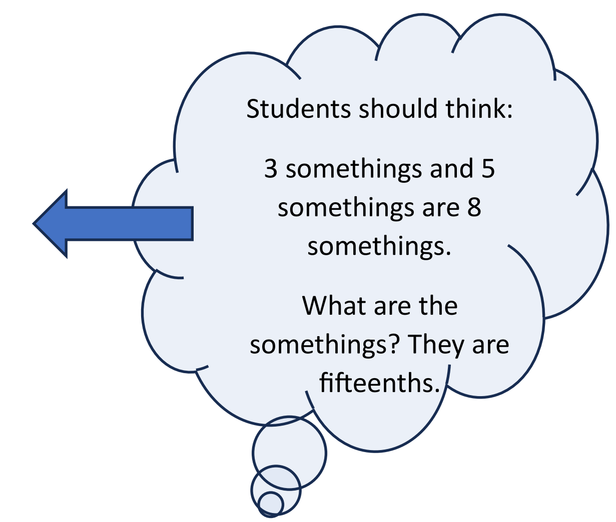
- from $1\frac{4}{5}$ to 2;
- then from 2 to 4;
- and from 4 to $4\frac{1}{3}$



Altogether: $\mathbf{4\frac{1}{3} - 1\frac{4}{5} = \frac{1}{5} + 2 + \frac{1}{3} = 2 + \left(\frac{1}{5} + \frac{1}{3}\right)}$

When adding the two proper fractions, students should employ strategies that work for them. (See section above for the grid method or see below for a symbolic example)

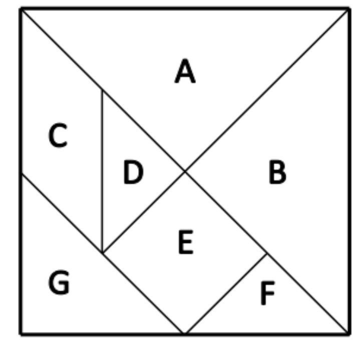
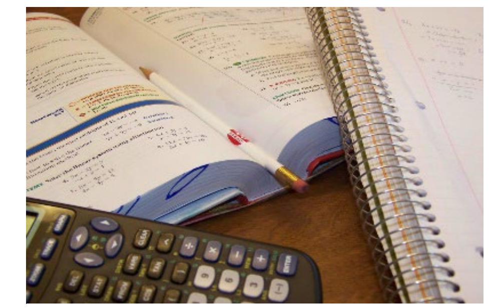
$$\frac{1}{5} + \frac{1}{3} = \frac{1 \times 3}{5 \times 3} + \frac{1 \times 5}{3 \times 5} = \frac{3}{15} + \frac{5}{15} = \frac{8}{15}$$



So, the final answer will be:

$$4\frac{1}{3} - 1\frac{4}{5} = 2\frac{8}{15}$$

Activities to Support Lesson Planning

Grade 6	Grade 7	Grade 8
Your cookie recipe calls for two and a half cups of flour. As a group, brainstorm various ways to measure out the correct amount using a set of measuring cups: 1 cup, $\frac{1}{2}$ cup, $\frac{1}{3}$ cup and $\frac{1}{4}$ cup. 	A tangram is a square puzzle that is divided into seven shapes. Based on the tangram below, answer the following questions with the understanding that A = $\frac{1}{4}$: 	You completed $\frac{1}{4}$ of your homework before lunch. And then $\frac{2}{3}$ of the remaining work after lunch. 
Knowledge: Based on the list your group created, which measuring cup is used the least?	**Knowledge:** Which shape represents half of A?	**Knowledge:** Do you have homework left to complete?
Application: You only have the $\frac{1}{4}$ cup. How would you measure out the amount needed?	**Application:** Which two tangram pieces add up to the value of B? Which three tangram pieces add up to the value of B?	**Application:** How many times more work did you do after lunch compared to before lunch?
Analysis: Describe a scenario where $\frac{1}{3}$ cup would be useful and decide whether the measurement with $\frac{1}{3}$ cup would be exact or an estimation.	**Analysis:** Invent a problem on your own and solve it.	**Analysis:** If the fractions of work completed were $\frac{1}{3}$ before lunch and $\frac{3}{4}$ after lunch: How are the solutions the same? How can they help you solve future problems?

Sample Questions for Assessment			
Cognitive Level	**Grade 6**	**Grade 7**	**Grade 8**
Knowledge	i) Write two improper fractions that are between 4 and 5. ii) What are the equivalent mixed numbers?	Calculate: $\frac{1}{8}+\frac{1}{2}=\_\_$ $\frac{12}{16}-\frac{3}{4}=\_\_$	i) What operation is being referenced when we say: "one third of three quarters"? ii) What operation can be represented as repeated subtraction?
Application	i) Choose a value close to $4\frac{1}{2}$. Represent it as both a mixed number and an improper fraction. ii) What is another value in between your chosen value and $4\frac{1}{2}$?	i) Your friend added two fractions and found the answer to be $\frac{5}{8}$. What could the fractions have been? Are they the only two possibilities? ii) Your friend subtracted two fractions and found the answer to be 0. The fractions had different denominators. What could the fractions have been? Are they the only two possibilities?	i) A recipe calls for $\frac{3}{4}$ cup of flour. In the second step, the instructions say to set aside $\frac{1}{3}$ of the measured flour to use later. How much flour (in cups) will be used now? ii) You are baking cupcakes, and you have 3 cups of cocoa powder. Each batch requires 1/4 cup of cocoa for the batter and 1/8 cup for the topping. How many full batches can you make?
Analysis	i) What is true for all mixed numbers between 4 and 5? ii) What is true for all improper fractions between 4 and 5?	Explain to your classmate why their answer is not reasonable. $\frac{5}{6}+\frac{5}{8}=\frac{10}{14}$ $\frac{10}{14}-\frac{5}{6}=\frac{5}{8}$	i) The original recipe calls for 3/4 cup of flour, and you are told to set aside 1/3 of that amount to use later. When you double the recipe, the amount set aside remains the same. How much flour (in cups) will be used now in the doubled batch? ii) Shamus has 3 cups of cocoa powder. Recipe A: each batch uses 1/4 cup of cocoa for the batter and 1/8 cup for the topping. Recipe B: each batch uses 3/8 cup of cocoa for batter. Topping recipe not included. Which recipe would you recommend to Shamus? Why?

Supporting Resources

References:

- Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2014). Mathematics 6 Curriculum Guide. Halifax, NS: Mathematics 6 Curriculum Guide (2014) pages 8 and 52–60.
- Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2015). Mathematics 7 Curriculum Guide. Halifax, NS: Mathematics 7 Curriculum Guide (2015) pages 8, 9, 67-78 and 88-93
- Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2015). Mathematics 8 Curriculum Guide. Halifax, NS: Mathematics 8 Curriculum Guide (2015) pages 14–15 and 73–89.
- Parrish, S., & Dominick, A. (2016). *Number talks. Fractions, decimals, and percentages: a multimedia professional learning resource*. Math Solutions. (pg 284)
- Reeves, D. B. (2004) *Making Standards Work: How to implement standards-based assessment in the classroom, school and district (3rd ed.)*. Advanced Learning Press. pp. 53
- Small, M. (2009). *Making math meaningful to Canadian students, K-8*. Nelson Education. (pg 217-218)

Manipulatives and models to support learning:
2 coloured counters, 10 frames, base ten blocks, open number lines, Cuisinaire rods, pattern blocks, and Tangrams.

Print:

- Small, M. Making Mathematics Meaningful to Canadian Students, K–8, Toronto, Ont. Nelson Education Ltd., 2009, pp 202
- Small, M. Making Mathematics Meaningful to Canadian Students, K–8, Second Edition, Toronto, Ont. Nelson Education Ltd., pp 249-277

- Van de Walle, J.A. and Lovin, L.H. Teaching Student-Centered Mathematics, Grades 5–8, Boston, Pearson Allyn & Bacon, 2006, pp 69
- Van de Walle, J.A. and Lovin, L.H. Teaching Student-Centered Mathematics, Grades 5-8, Volume Three, Boston, Pearson Allyn & Bacon, 2006, pp 93–130

- Garneau et al. Math Makes Sense 7, Pearson, 2007 - Unit 3: Fractions, Decimals and Percents (NSSBB #: 2001640) – Section 3.2 Comparing and Ordering Fractions and Decimals
- Garneau et al. Math Makes Sense 7, Pearson, 2007 - Unit 5: Operations with Fractions (NSSBB #: 2001640)
- Barron et al. Math Makes Sense 8, Pearson, 2008 (NSSBB #: 2001642) - Unit 3: Operations with Fractions

Digital:

- “Cuisenaire Environment,” NRICH Enriching Mathematics (University of Cambridge 2015): http://nrich.maths.org/4348
- “Fraction Pieces,” Utah State University (Utah State University 2015): http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities
- “iTools: Fractions,” Houghton Mifflin Harcourt School Publishers (Houghton Mifflin Harcourt School Publishers 2015): www.k6.thinkcentral.com/content/hsp/math/hspmath/na/common/itools_int_9780547584997_/fraction s.html
- “Mathematics Blackline Masters Grades P to 9, Table of Contents,” Nova Scotia Department of Education and Early Childhood Development (Province of Nova Scotia 2015): http://lrt.ednet.ns.ca/PD/BLM/table_of_contents.htm
- “Modeling Fractions with Cuisenaire Rods,” PBS Learning Media (PBS and WGH 2015): www.pbslearningmedia.org/resource/rttt12.math.cuisenaire/modelling-fractions-with-cuisenairerod

- “Equivalent Fractions,” Illuminations: Resources for Teachers (National Council of Teachers of Mathematics 2015): http://illuminations.nctm.org/Activity.aspx?id=3510

- “Virtual Manipulatives: Fractions, Decimals, Percents,” abcya.com (ABCya.com LLC 2015): http://media.abcya.com/games/fraction_tiles/flash/fraction_tiles.swf
- “Equivalent Fractions,” Illuminations: Resources for Teaching Math (National Council of Teachers of Mathematics 2015): http://illuminations.nctm.org/Activity.aspx?id=3510